2. Physics Modeling-based Data Analysis

This chapter introduces the process of finding solution and modeling analysis of the experimental data using Excel Solver, which is the add-in tool.

2.1

Excel Solver

In chapter 1, the way of mathematical analysis for the experimental data using Excel was introduced. In chapter 2, a higher level experiment analysis, which uses physics modeling in Excel, will be explained. Physics modeling can be used not only in the theoretical analysis, which calculates algebraic relationship in an equation of motion and finds solutions, but also in the analysis of the experimental data using the prediction of theoretical model according to the physical circumstances. In 2.1, as a process of physics modeling, using Excel Solver[1], we will be introduced the way to find mathematical solutions satisfying limited conditions in a certain observation rage.

---

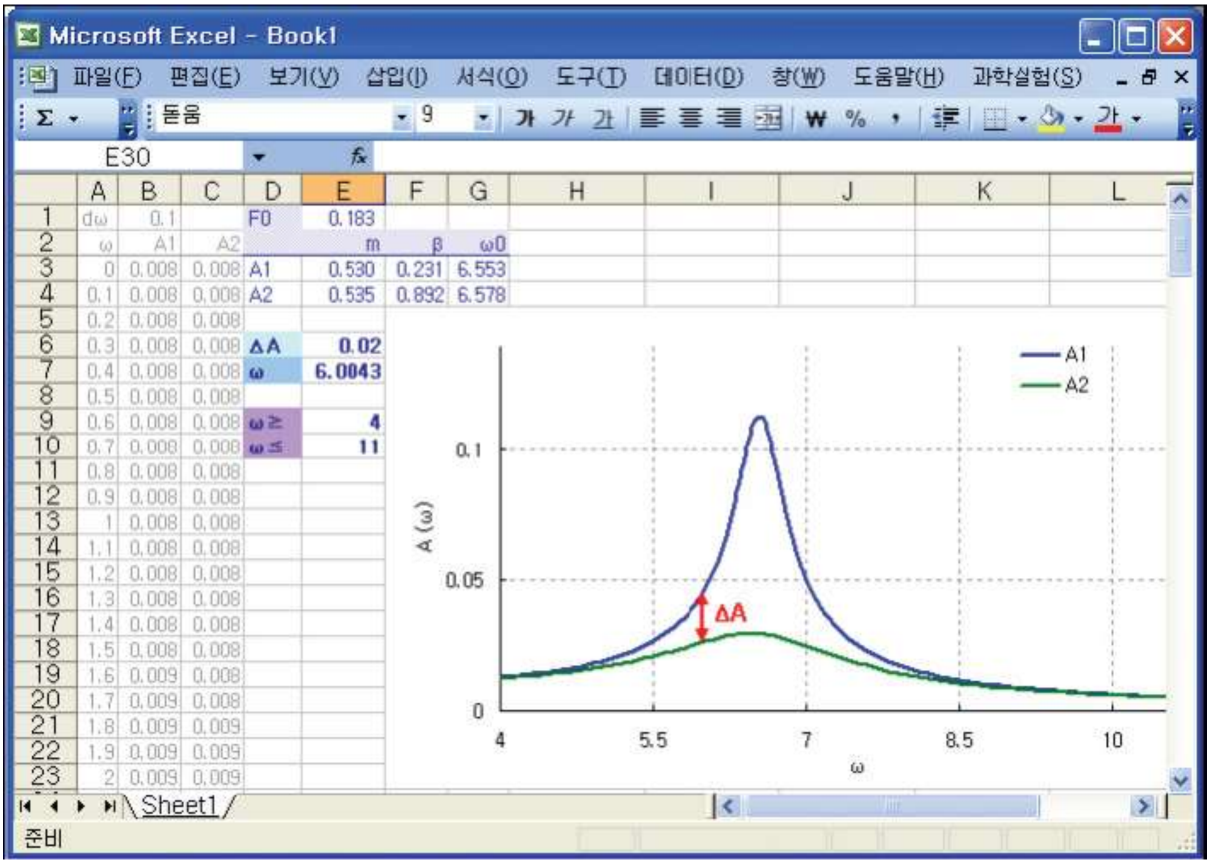
[1] Excel Solver can be used both in the physics modeling-based theoretical prediction and experiment analysis. This function is the add-in tool of Excel.

2.1.1 Excel Solver for Physics Model

With examples, let's learn about how to find solutions using mathematical models of physical phenomena. Picture 2.1.1 is the graph of the amplitude $A(\omega)$'s prediction model $A(\omega)-\omega$, which is based on the results of the forced vibration experiment in chapter 5 and which is about the two experiment A_1 and A_2. When the target cell 's mathematical model is as below,

$$A(\omega)=\frac{F_0/m}{(\omega_0^2-\omega^2)^2+4\beta^2\omega^2}$$

Then, calculate the value of ω to make $\Delta A = A_2 - A_1 = 0.02$. Input the formula of ΔA in [Target Cell] $E$6, and the variable ω in [Changing Cell] $E$7.



Picture 2.1.1 finding solution with physics prediction model: example of calculating the value of ω that makes $\Delta A = 0.02$

	D	E	F	G
1	F0	0.183		
2		m	β	ω0
3	A1	0.530	0.231	6.553
4	A2	0.535	0.892	6.578
5				
6	ΔA	0.02[2]		
7	ω	6.00[3]		
8				
9	ω>=	4		
10	ω<=	11		

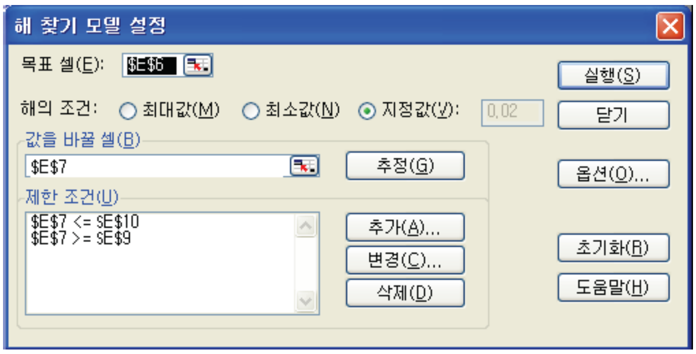
23

Table 2.1.1 example of Excel worksheet design for the model analysis[4]

When you execute the Excel Solver with setting up ΔA's [Excel Solver]-[Solution Condition] in [Excel Solver]-[Designated Value], you can calculate the value of ω. AS this, practice the process of setting up mathematical models for various physical circumstances and finding solutions.

[2] Formula of $\Delta A = A_1 - A_2$ is “=+ ($E$1/$E$3)/SQRT(($G$3^2-E7^2)^2+ 4*$F$3^2*E7^2)-($E$1/$E$4)/SQRT(($G$4^2-E7^2)^2+ 4*$F$4^2*E7^2)”.

[3] Before executing [Excel Solver], predict the approximate value of the solution and write it in the cell.



[4] [Limited Condition] 4<=ω and <=11 is written in cell $E$9 and $E$10, and [Solution Condition] ΔA should be set up in [Designated Value].

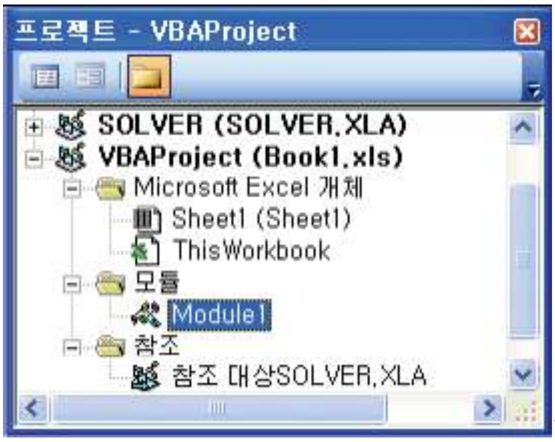
2.1.2 Excel Solver Questions

Excel solver questions can write VBA macro code of Excel[5] and record it in VBA command button and be executed repeatedly[6] according to various physics models. As in 2.1.1, set up physics model formulas for Excel solver questions as below and calculate mathematical solutions.

1. On the horizontal surface, set up a model for a cart's velocity which gets force in proportion to the velocity. Then when the initial velocity is $v_0 = 0.453\mathrm{m/s}$ and after 0.5 second, calculate the ratio of the velocity according to the cart's mass 0.525kg, 1.052kg.

2. In the collision of a cart by a magnet bumper, when the maximum of the force during the collision is 3.525N, calculate the time t that makes the impulse (t) 50%. It should be done in F-t graph with the maximum value of F(t) as standard and by Gaussian collision model[7] within FWHM's time range ±0.0325s.

3. When a cart that is doing damping oscillation has natural frequency 1.051Hz and Amplitude A_0=0.119m, and when the phase is 0 or 2π, calculate the damping constant β that makes $\Delta x = |x_0 - x_{2\pi}| = 0.001$ in the graph of velocity and location $\dot{x} - x$.



[5] After recording [Excel Solver] macro in[Tool]-[Macro]-[New Macro Recording], use VBA code of macro function "Sub Macro1()", which has been written automatically in "Module 1" of [VBA]-[Module], in procedure of [CommandButton]. To use VBA cod of the module, you should choose to use "SOLVER" in [VBE]-[Tool]-[Reference].

[6] For example, this is useful when you calculate solution by changing [Limited Condition], which is the constraints condition of finding solutions in physics models that have same mathematical process of solution finding such as [Target Cell], [Changing Cell] or [Solution Condition].

[7] Refer to Chapter 4. Collision.

2.2

Modeling Analysis

Modeling Analysis is a process of setting up the mathematical model for physical circumstances and analyzing real experimental data based on the expectation about the model. The process of modeling-based data analysis in Excel is as follows.

a. Collect experimental data, and extract[1] the data within the analysis range according to the experiment situations.
b. Define the modulus of the physical model's formula to [Tool]-[Excel Solver]-[Value Changing Cell] and write down the predicted data series of the model in new rows of worksheet.
c. Calculate the difference between the experiment data series and the predicted data series of the model in the new data series and set up the target cell.
d. Set up [Tool]-[Excel Solver]-[Solution Condition] as [Minimum][2] and execute the process to calculate the modulus of model's formula.

[1] When there is data irrelevant to the experiment, delete it. For example, the values which are collected before and after the motion are irrelevant to the experiment so you can delete them. The easy way to extract data is either to copy the needed data as block and paste it to the new rows of the sheet or to write and use VBA which extracts data automatically.
[2] [Solution Condition] can be set up differently according to the physics modeling situations.

2.2.1. Data Analysis according to Index Prediction Model

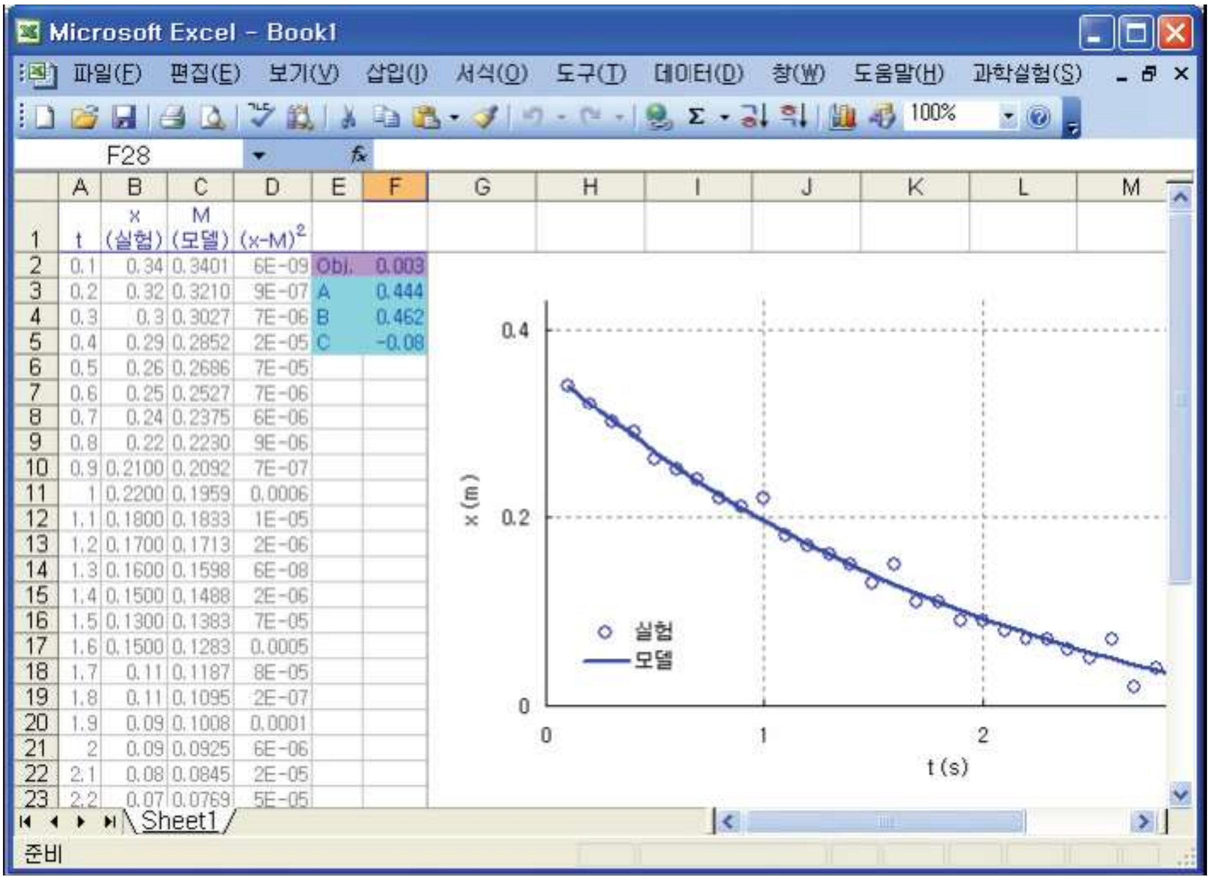
In chapter 1, linear trend analysis and index trend analysis in $x-t$, $v-t$ graphs were explained. Here, with the same experiment, physics modeling-based analysis will be done.

a. In picture 2.2.1, row C is the experiment data series x that will be analyzed, and row D is the predicted data series according to the model's formula.
b. Set up cell F2 as target cell and input formula "=+SUM(D2:D2000)".
c. Execute [Excel Solver][3] and calculate the modulus of model's formula A, B, and C in cell F3, F4 and F5, which are defined as [Value Changing Cell].

	A	B	C	D
	t	x	M (모델)	$(x-M)^2$
2	0.1	0.34	=+$F$3*EXP(-$F$4*A2)+$F$5	
3	0.2	0.32	=+$F$3*EXP(-$F$4*A3)+$F$5	
4	0.3	0.30	=+$F$3*EXP(-$F$4*A4)+$F$5	
…				

Table 2.2.1 example of Excel design for the modeling analysis

[3] In [Excel Solver], the analysis can be done with changing [Limited Condition] and [Option] according to the situations. If you click [Execution] Button, the value of target cell will return the optimum value that fits the [Solution Condition] into [Value Changing Cell].



Picture 2.2.1 $x-t$ graph after physics modeling-based analysis

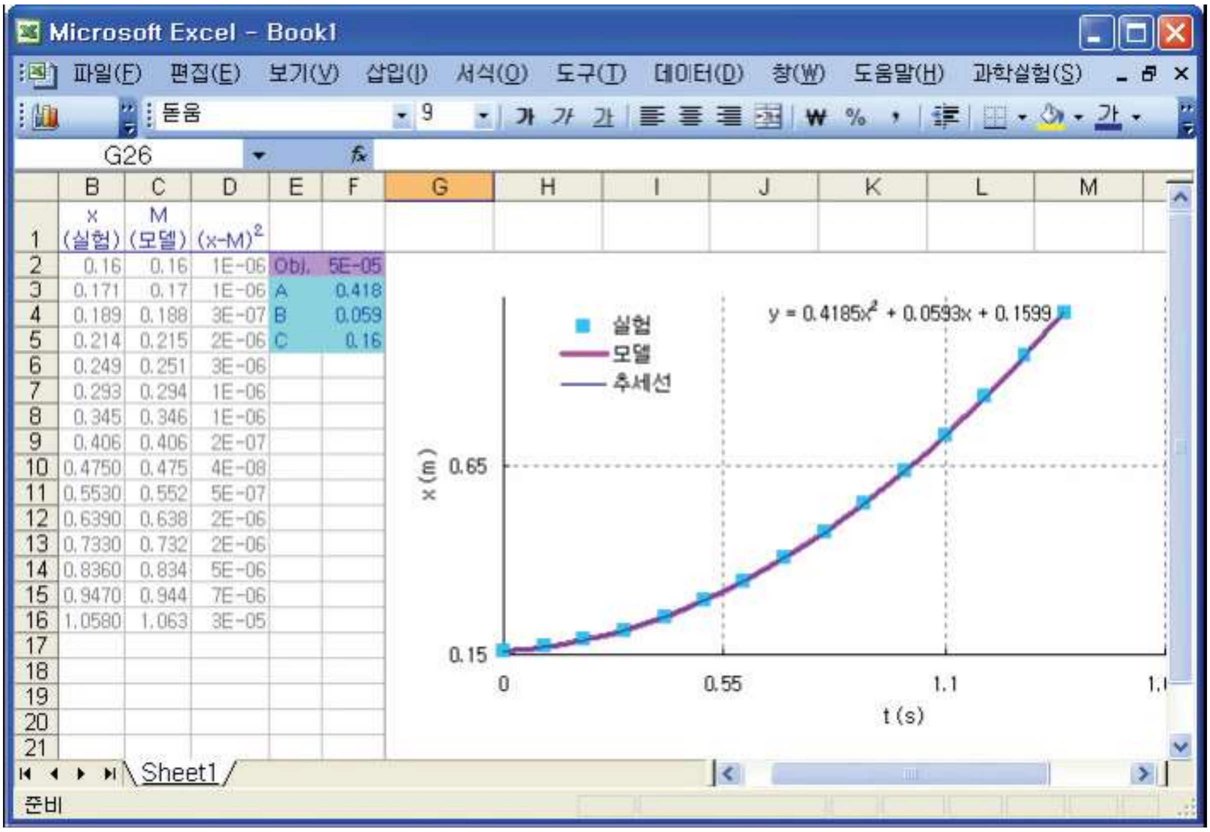
Picture 2.2.1 is the $x-t$ graph, which is the result of the physics model's formula $Ae^{(-Bt)}+C$. The formula's modulus will be from F3 to F5, which are recorded in [Value Changing Cell][4], and it will be $0.443e^{(-0.462t)}-0.08$. Next exercise is an example of physics modeling-based analysis.

Exercise 2.2.2. Polynomial Prediction Model

Picture 2.2.2 is the result of a cart's uniformly accelerated motion on a straight track. The formula of the experiment's result is $0.418t^2+0.059t+0.16$ and this formula is calculated by setting up At^2+Bt+C as the model formula of polynomial prediction and finding the modulus.

---

[4] Input the approximate value of the predicted modulus and execute [Excel Solver], then the optimum modulus will be returned in [Solution Condition] cell F2.



Picture 2.2.2 $x-t$ graph which is the result of data analysis according to the polynomial prediction model[5]: the target cell's value is $F$2=5E-O5≈0, so you can see that the result of modeling experiment analysis is optimum.

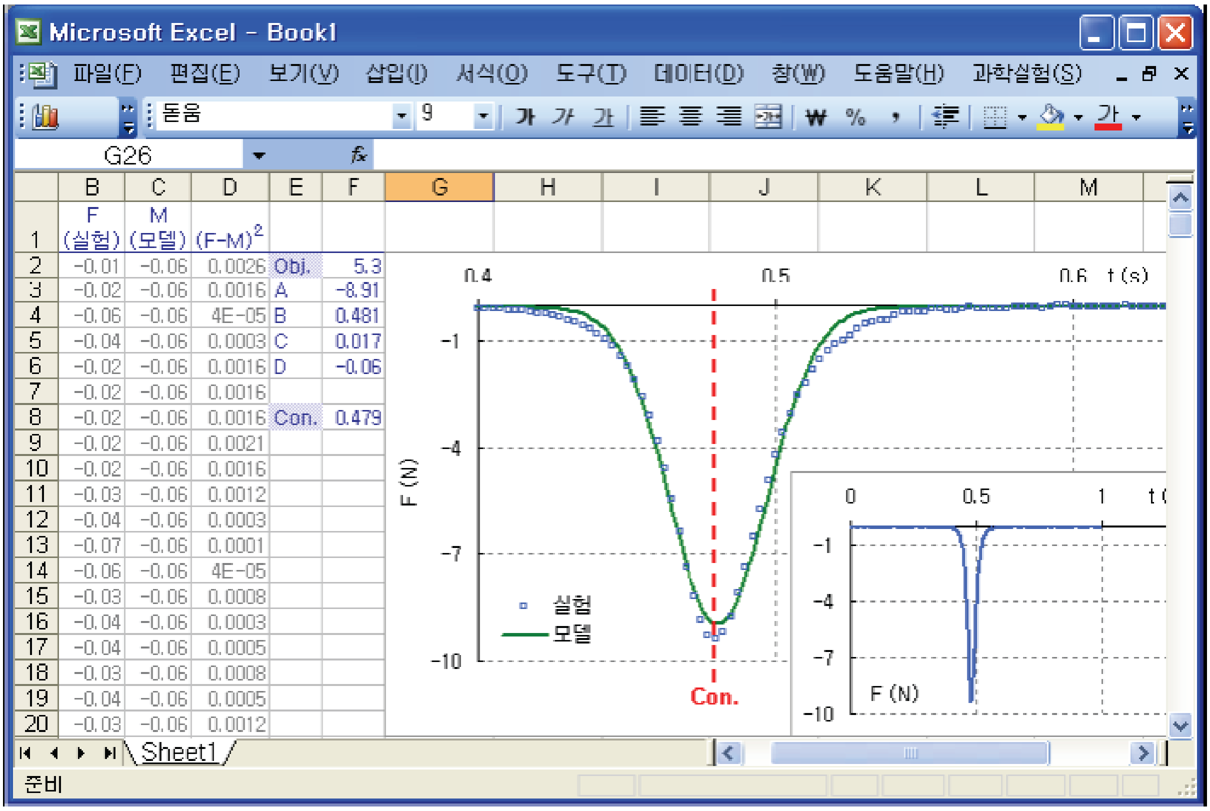
Exercise 2.2.2: Gaussian Prediction Model

Picture 2.2.3 is F-t graph's result, which is the result of an experiment in which a cart is driven back because of a magnet bumper. Within the range from $-\sigma$ to $+\sigma$ and about time t, the prediction model formula which is near to the Gaussian curve is $Ae^{-(t-B)^2/2C^2}+D$, and the result formula of finding modulus is $-8.906e^{[-(t-0.481)^2/2(-0.017)^2]}-0.062$. According to the model analysis result, $\sigma=-0.016$ so FWHM[6]=-0.019.

---

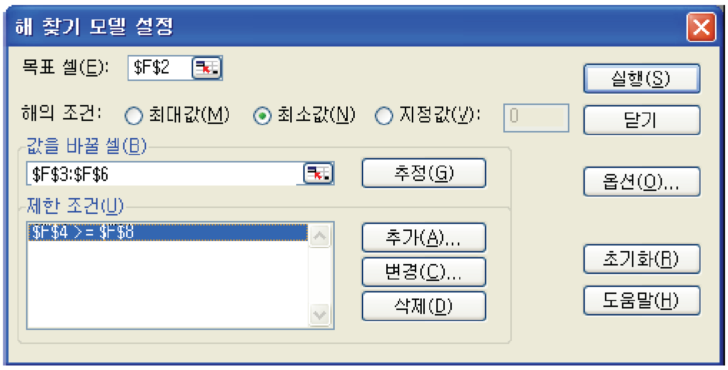
[5] The $x-t$ graphs of the experiment data series, of the polynomial model's analysis result, and of the trend line's analysis results correspond to one another. By practicing this polynomial model, make a challenge to more complex modeling analysis.

[6] FWHM(full width at half maximum): read Chapter 4. Collision.



Picture 2.2.3 $x-t$ graph which is the result of data analysis according to the Gaussian prediction model[7]

The graph of picture 2.2.3 is the result of setting up modulus B value[8] as the limited condition and analyzing. Record the time value (X axis) (which is the maximum value of $|F(Y\ axis)|$) in cell F8, and set up the value of F8 as [Limited Condition] in [Excel Solver]. Set up the Value of F4(which is the modulus B of model formula) as [Cell Reference Area] and click [Execution] button, then the optimum value will be returned in [Value Changing Cell] according to the calculation ways that was set up in [Option].

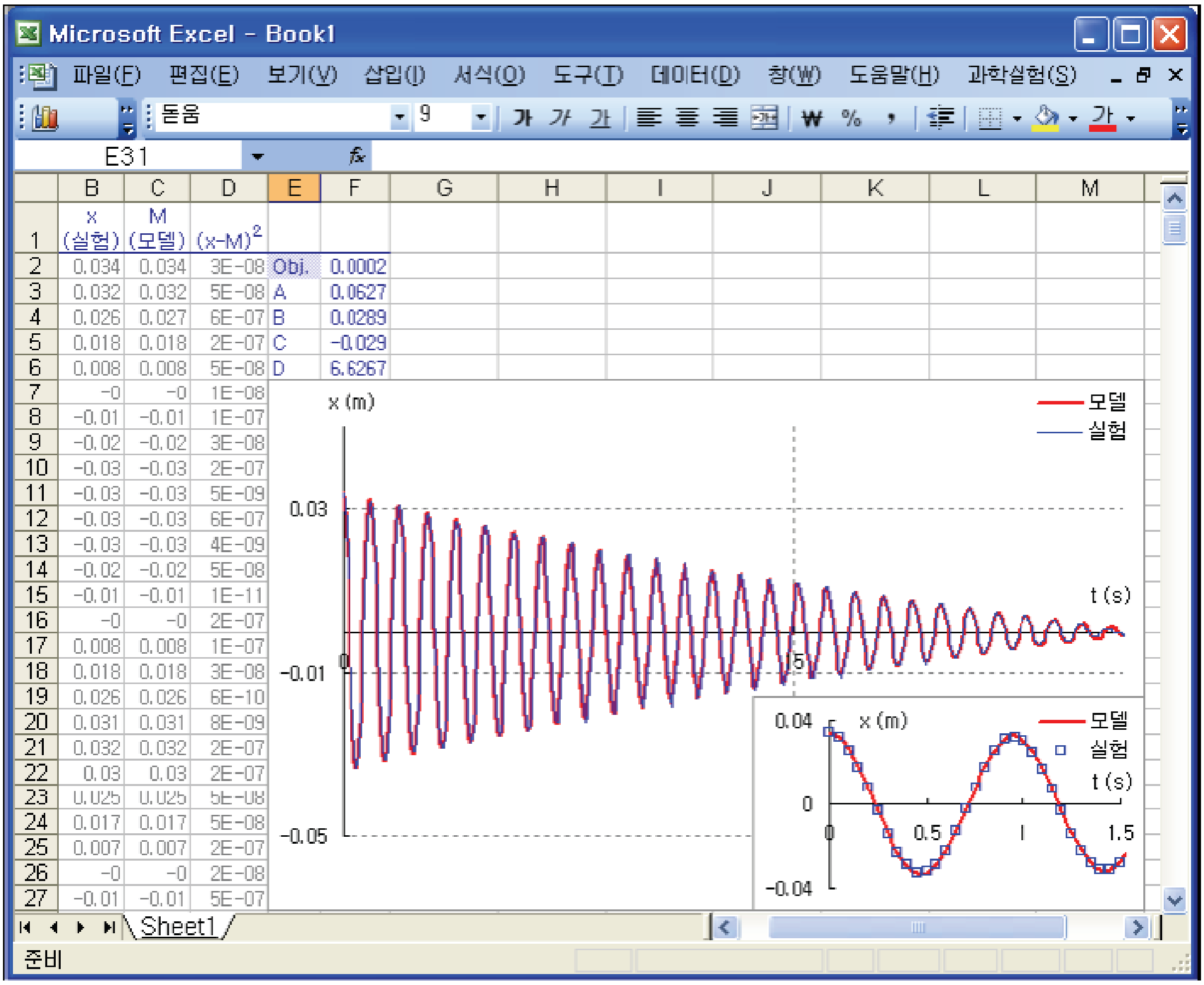


[7] [Value Changing Cell] is the $F$3:$F$6 range in which the formula's modulus that row C refers to is recorded. The Target cell $F$2 is the cell that records the cell's condition in which the sum of row D's values is minimum.

[8] In $x-t$ graph, read the time(X axis) of $|F(Y\ axis)|$'s maximum value with the mouse cursor and record it in B value.

2.2.2. Prediction Model and Data Analysis of Periodical Damping

Continuing from general index prediction model, in 2.2.2, the way of data analysis with the prediction model of a periodical damping will be introduced. Picture 2.2.4 is the result of executing curve fitting[9] for $x-t$ graph's exponential damping with periodical damping prediction model's formula$[Ae^{-Bt}+C]\cdot\cos(Dt)$. Moculus A is the maximum amplitude of the damped oscillation, B is the damping constant, and D is the angular frequency.

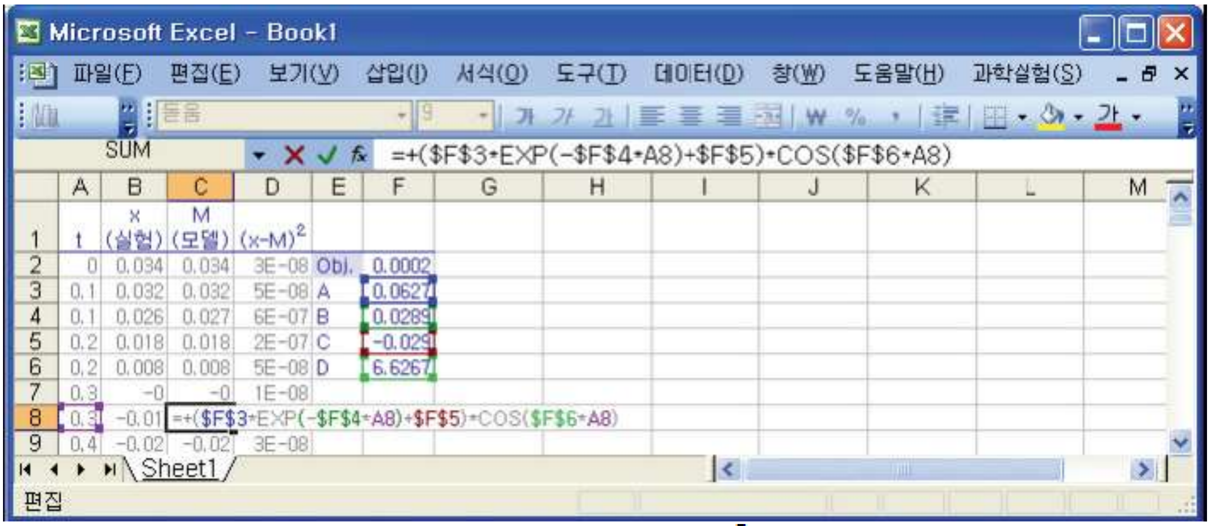


Picture 2.2.4 $x-t$ graph which is the result of data analysis according to the periodical damping prediction model: you can see that the graph according to the model and the graph of experiment data series are correspond to each other.

[9] Refer to Chapter 5. Oscillation for the curve fitting. In Chapter 5. Mathematical curve fitting using VBA is introduced.

The periodical damping model is susceptive according to the solution finding situations, so predict the approximate value of the solution and input the modulus and limited conditions for the model formula properly. The process of modeling analysis is as follows.

a. In picture 2.2.4, row C is the experiment data series x to be analyzed and row D is the prediction data series according to the exponential damping model's formula. Input the formula of data series as picture 2.2.5.



Picture 2.2.5 inputting formula of model data series: $[Ae^{-Bt} + C] \cdot \cos(Dt)$

b. Set up cell F2 as target cell and input the formula "=+SUM(D2:D2000)"[10]

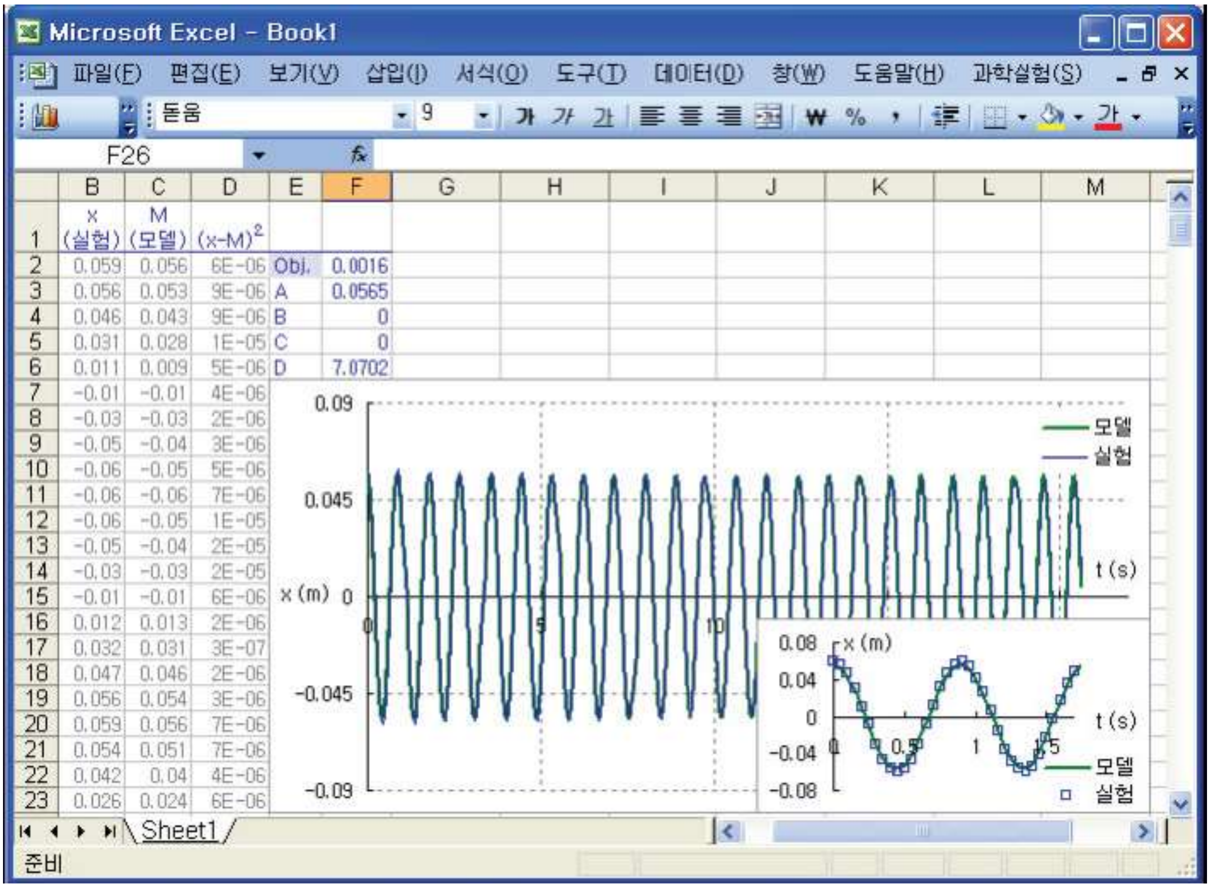
c. Execute [Excel Solver][11] and calculate the modulus of model formula A, B, C and D in cell F3, F4, F5 and F6, which are defined as [Value Changing Cell].

[10] This is the solution's condition that makes the difference between the experiment data series (row C) and the model data series(row D) minimum.

[11] In [Excel Solver], the analysis can be done with changing [Limited Condition] and [Option] according to the situations. If you click [Execution] Button, the value of target cell will return the optimum value that fits the [Solution Condition] into [Value Changing Cell]

Exercise 2.2.3: Periodic Function Prediction Model

Picture 2.2.6 is $x-t$ graph which is the result of an oscillation with a ball and a spring and the result of analysis according to the formula model Acos(Dt). As a result, the formula is 0.057cos(7.070t). The angular frequency is $\omega = 7.070$, so the frequency is $f = 1.125\text{Hz}$.



	B	C	D	E	F
1	x (실험)	M (모델)	$(x-M)^2$		
2	0.059	0.056	6E-06	Obj.	0.0016
3	0.056	0.053	9E-06	A	0.0565
4	0.046	0.043	9E-06	B	0
5	0.031	0.028	1E-05	C	0
6	0.011	0.009	5E-06	D	7.0702
7	-0.01	-0.01	4E-06		
8	-0.03	-0.03	2E-06		
9	-0.05	-0.04	3E-06		
10	-0.06	-0.05	5E-06		
11	-0.06	-0.06	7E-06		
12	-0.06	-0.05	1E-05		
13	-0.05	-0.04	2E-05		
14	-0.03	-0.03	2E-05		
15	-0.01	-0.01	6E-06		
16	0.012	0.013	2E-06		
17	0.032	0.031	3E-07		
18	0.047	0.046	2E-06		
19	0.056	0.054	3E-06		
20	0.059	0.056	7E-06		
21	0.054	0.051	7E-06		
22	0.042	0.04	4E-06		
23	0.026	0.024	6E-06		

Picture 2.2.6: $x-t$ graph which is the result of data analysis according to cos function prediction model: you should find solution with setting up the model as Acos(Dt), cell F3(value of modulus A) and cell F6(value of modulus D) as [Value Changing Cell].

In picture 2.2.6, input [Value Changing Cell] as "$F$3, $F$6", and before finding solution, input the predicted value of the angular frequency[12] in cell F6.

[12] For example, if the value is 7.070 after the analysis, input value near 7.070 before finding solution. To find out how much the prediction deviates, you should set up [Interim Findings Checkout] in [Excel Solver Model Setup]-[Option] and check out the calculated value changes near the solution as the calculation is repeated.